ANALYSIS OF THE NONEQUILIBRIUM FLOW PARAMETERS IN A GAS-DISCHARGE CHAMBER

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The approximate analytic solution of equations describing the subsonic gas flow in a multielectrode chamber with a glow discharge is obtained.

Coefficients whose values are determined experimentally [1] are used in numerical computations of the nonuniform flow parameters in a gas-discharge chamber (GDC). Among them are all the GDC limit characteristics (the highest currents, pressures, etc.) as well as the efficiency of the process for exciting molecule vibrations. In its turn, the experiments are analyzed within the framework of some kind of computational model [2]. Possibly more simple analytic solutions are preferred here. They are also useful to check numerical computation programs and in GDC design. A solution in quadratures is presented below for the simplified problem of the subsonic gas flow parameters in a multielectrode chamber with a transverse discharge and certain particular cases are examined when the solution can be represented in the form of a combination of elementary functions.

The fundamental geometric parameters of a multielectrode GDC are the channel height (interelectrode gap), the area of its cross section and the spacing between cathodes (or anodes) in the gas flow direction. Values of the static pressure at points distributed along the channel length, of the currents of each series of electrodes arranged in one cross section, the average voltages and gasdynamic flow parameters at the input for these series of electrodes are ordinarily measured in experiment. We shall consider the listed geometric parameters and the quantities measured in tests as well as the values of the near-electrode potential drops to be given in our computations. Let us note that the magnitudes of the near-electrode potential drops are functions of the coordinates and the other GDC parameters [3-6].

The physical processes occurring in a glow discharge will be considered within the framework of the simplified model described in [7]. In the near-cathode region the charge is transferred mainly by positive ions [3]. In the near-anode region of the discharge apparently ionic conductivity also predominates for significant electronegative ion concentrations [8]. It can hence be considered that the power of the near-cathode and near-anode sections of the discharge is expended in heating the electrodes and the adjoining gas layer. Moreover, by virtue of the smallness of the magnitudes of the characteristic near-electrode domain scales and the smallness of the flow rates of the near-wall gas layers, the characteristic length of vibrational energy relaxation should be small, which would contribute to the transformation of this energy into thermal energy. For small concentrations of electronegative ions in the near-anode domain, the magnitude of the anode drop is small (~10 V [9]), and its contribution to the GDC energy balance is insignificant. All this permits us to consider that excitation of molecule vibrations occurs only in the positive column (PC) of the discharge. A certain fraction of the PC power is expended on this. It can be considered that the rest of the PC power goes into heating the gas which occurs practically instantaneously [2]. Heat liberation results in acceleration of the gas flow. Such a flow possesses the property of equilibration of the velocity diagram [10], which permits consideration of the problem in a one-dimensional formulation. The flow parameters are described in the mentioned approximation by a system of equations of state, continuity, vibrational energy, and energy of the gas molecule translational and rotational degrees of freedom, from which the expressions

$$\frac{p_0 s_0 v_0}{T_0}\,\frac{dw(x)}{s(x)\,dx} = \frac{\xi_1 R}{\mu}\,jE(x) - \frac{w(x)\,p^2(x)}{\langle p\tau\rangle\,T(x)}\,, \tag{1}$$

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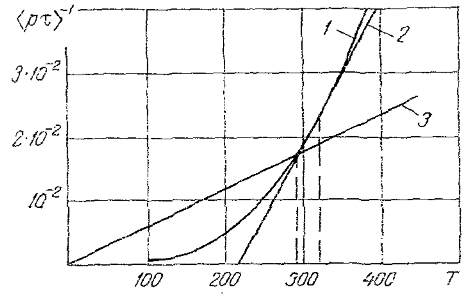

Fig. 1. Temperature dependence of the reduced relaxation time constant $\langle p\tau\rangle$ of the vibrational excitation of nitrogen in a gas mixture: 1) the function f(T) under conditions of the experiment [2] for 1.5% air humidity; 2) the approximation $f(T) \approx k_2T - D$; 3) $f(T) \approx k_1T$, $\langle p\tau\rangle^{-1}$, $\text{Pa}^{-1}\cdot\text{sec}^{-1}$; T, °K.

$$\frac{p_0s_0v_0}{T_0}\,\frac{c\,dT(x)}{s(x)\,dx}=\frac{R}{\mu}\,jE(x)\left[1-\xi_1+\xi_2\,\frac{U_a(x)+U_{\text{к}}(x)}{U(x)}\right]+\frac{w(x)\,p^2(x)}{\langle p\tau\rangle\,T(x)}\,. \tag{2}$$

can be obtained [7]. Here $\langle p\tau\rangle^{-1} = f(T)$ is a certain known function of the temperature. The power density in the positive column can be expressed as

$$jE(x)=[U(x)-U_a(x)-U_{\text{к}}(x)]\,j(x)/h(x). \tag{3}$$

After some manipulation, we find from (1) and (2)

$$w(x)+cT(x)=\frac{T_0R}{p_0v_0s_0\mu}\int\left[1+\xi_2\,\frac{U_{\text{к}}(x)+U_a(x)}{U(x)}\right]jE(x)\,s(x)\,dx+w_0+cT_0\,; \tag{4}$$

as a rule f(T) is an increasing function (Fig. 1). Hence, we can take $\langle p\tau\rangle^{-1} = k_1T$, where $k_1 \approx f(T_0)/T_0$. Then integrating (1) yields

$$w^{(1)}(x)=\left[w_0+\frac{T_0R}{\mu p_0v_0s_0}\int\frac{\xi_1jE(x)\,s(x)\,dx}{\exp\left[-\dfrac{T_0k_1}{p_0v_0s_0}\displaystyle\int p^2(x)\,s(x)\,dx\right]}\right]\exp\left[-\frac{T_0k_1}{p_0v_0s_0}\int p^2(x)\,s(x)\,dx\right]. \tag{5}$$

We determined T(x) in a first approximation from (4) and (5)

$$T^{(1)}(x)=T_0+\frac{1}{c}\left[w_0+\frac{T_0R}{p_0v_0s_0\mu}\int\left[1+\xi_2\,\frac{U_{\text{к}}(x)+U_a(x)}{U(x)}\right]jE(x)\,s(x)\,dx-w^{(1)}(x)\right]. \tag{6}$$

In a second approximation $w^{(2)}(x)$ and $T^{(2)}(x)$ are found as follows. The function $\langle p\tau\rangle^{-1} = f(T)$ is written explicitly in (1) and (2). It depends on the molar fractions of the gas mixture components and the reduced binary relaxation times, whose temperature dependence is known [11]. Using its approximation $T^{(1)}(x)$ for T(x) we find $w^{(2)}(x)$ from (1). Furthermore, we determine $T^{(2)}(x)$ from (4). Subsequent approximations can also be obtained analogously.

The appropriate flow velocities and the volume densities of the vibrationally excited molecules (one-quantum excitation) are determined from the formulas

$$v^{(n)}(x)=p_0v_0s_0T^{(n)}(x)/T_0p(x)\,s(x),\quad N^{(n)}(x)=\mu p(x)\,w^{(n)}(x)/\delta ERT^{(n)}(x). \tag{7}$$

If a vibrationally excited gas proceeds from the GDC into an apparatus downstream, then its efficiency equals

$$\eta^{(n)}(x)=w^{(n)}(x)/[w^{(n)}(x)+c\,(T^{(n)}(x)-T_0)]. \tag{8}$$

Let us examine particular cases of the solution obtained. If the constants s, p, $[U_k(x) + U_a(x)]/U(x) = C$, and the mean value jE, equal to

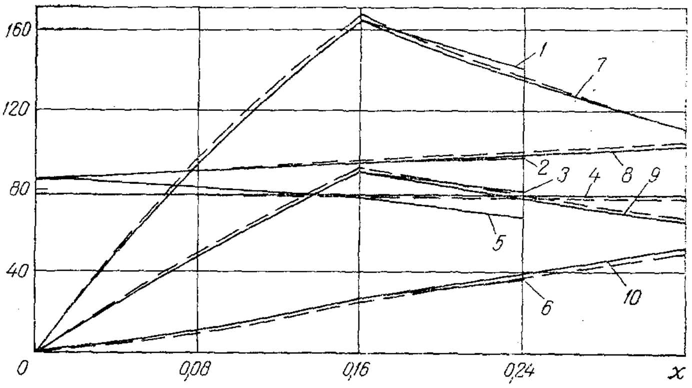

Fig. 2. Computation of the nonequilibrium flow parameters in a gas-discharge chamber (x < 0.16 m) and outside (x > 0.16 m) under experiment conditions [2]. Solid curves are a computation using (9) and (10): 1) $N^{(1)}$, 10^{21} m^{-3}, 2) $v^{(1)}$, $m \cdot sec^{-1}$, 3) $w^{(1)}$, 10^3 $J \cdot kg^{-1}$, 4) p, 10^2 Pa, 5) $\eta^{(1)}$, %, 6) $(T^{(1)} - T_0)$, °K, 7) $N^{(2)}$, 10^{21} m^{-3}, 8) $v^{(2)}$, $m \cdot sec^{-1}$, 9) $w^{(2)}$, 10^3 $J \cdot kg^{-1}$, 10 $(T^{(2)} - T_0)$, °K, Dashes are a digital computer computation by the method in [1], x, m.

$$\overline{jE} = \left[\sum_{i=1}^{q} I_i (U_i - U_{\text{к}i} - U_{\text{a}i})\right] / sq\Delta,$$

and $\langle p\tau \rangle^{-1} = k_1 T$, are considered, then (5), (6), (7), and (8) have the following form in a first approximation

$$\begin{gathered} w^{(1)}(x) = w_0 \exp(-\lambda x) + B[1 - \exp(-\lambda x)], \\ T^{(1)}(x) = T_0 + Ax - (B - w_0)[1 - \exp(-\lambda x)]/c, \\ \eta^{(1)}(x) = [w_0 \exp(-\lambda x) + B[1 - \exp(-\lambda x)]]/(cAx + w_0), \\ N^{(1)}(x) \sim w^{(1)}(x)/T^{(1)}(x). \end{gathered} \tag{9}$$

Here $B = \xi_1 \overline{jE} R/\mu p_0^2 k_1$, $A = [1 + \xi_2 C] T_0 R \overline{jE}/p_0 v_0 c\mu$, $\lambda = T_0 k_1 p_0 / v_0$.

If a nonexcited gas goes into the GDC, then it should be considered that $w_0 = 0$. If jE = 0 in the domain under consideration (outside the chamber or between chambers), then $w_0 \neq 0$ and A = B = 0. In the latter case the second approximation relaxation formulas have the form

$$w^{(2)}(x) = w_0 \exp\left\{-\frac{T_0 p_0}{v_0}\left[\left(k_2 - \frac{cD}{cT_0 + w_0}\right) x - \frac{cD \ln|1 + (1 - \exp(-\lambda x)) w_0/cT_0|}{\lambda(cT_0 + w_0)}\right]\right\},$$

$$T^{(2)}(x) = T_0 + [w_0 - w^{(2)}(x)]/c. \tag{10}$$

Here $k_2 = df(T)/dT$ for $T = T_0$, $D = k_2 T_0 - F(T_0)$ (Fig. 1).

As investigations show [5, 6], jE can vary significantly along the GDC length. If Δ > h, then the electric field in the GDC space is inhomogeneous. For $\Delta \leqslant h$ the electric field is comparatively homogeneous in the major portion of the GDC space, but the electron gas density distribution remains sharply modulated [6]. Hence, the current densities j(x) and jE(x) [see (3)] should also be considered sharply modulated. We take the following model of the GDC parameters: $s = \text{const}$, $p = \text{const}$, $\langle p\tau \rangle^{-1} = k_1 T(x)$, $[U_{\text{a}}(x) + U_{\text{к}}(x)]/U(x) = C = \text{const}$, a $jE(x) = jE_0 \sin^{2m}(\pi x/\Delta)$.

Normalizing jE_0 according to the formula

$$\frac{1}{q\Delta} \int_0^{q\Delta} jE_0 \sin^{2m}(\pi x/\Delta)\, dx = \overline{jE}$$

yields $jE_0 = \overline{jE}(2m)!!/(2m-1)!!$. In this case (5) and (6) have the following form for m = 1

$$w^{(1)}(x) = w_0 e^{-\lambda x} + B_0 \frac{2\pi^2(1-e^{-\lambda x}) + \left(\lambda^2\Delta^2 \sin\frac{\pi x}{\Delta} - \lambda\Delta 2\pi\cos\frac{\pi x}{\Delta}\right)\sin\frac{\pi x}{\Delta}}{\lambda^2\Delta^2 + 4\pi^2}, \tag{11}$$

$$T^{(1)}(x) = T_0 + A_0\left(\frac{x}{2} + \frac{\Delta\sin 2\frac{\pi x}{\Delta}}{\pi}\right) + \frac{w_0 - w^{(1)}(x)}{c}.$$

For m = 2 we obtain

$$w^{(1)}(x) = w_0 e^{-\lambda x} + \frac{B_0}{(\lambda^2\Delta^2 + 4\pi^2)(\lambda^2\Delta^2 + 16\pi^2)}\left[24\pi^4(1-e^{-\lambda x}) + \right.$$

$$\left. + (\lambda^2\Delta^2 + 4\pi^2)\left(\lambda^2\Delta^2\sin\frac{\pi x}{\Delta} - 4\pi\Delta\lambda\cos\frac{\pi x}{\Delta}\right)\sin^3\frac{\pi x}{\Delta} + 12\pi^2\left(\lambda^2\Delta^2\sin^2\frac{\pi x}{\Delta} - 2\pi\Delta\lambda\sin\frac{\pi x}{\Delta}\cos\frac{\pi x}{\Delta}\right)\right], \tag{12}$$

$$T^{(1)}(x) = T_0 + A_0\left[\frac{3x}{8} - \frac{\Delta}{4\pi}\left(\sin^3\frac{\pi x}{\Delta} + \frac{3}{2}\sin\frac{\pi x}{\Delta}\right)\cos\frac{\pi x}{\Delta}\right] + \frac{w_0 - w^{(1)}(x)}{c}.$$

The coefficients B_0 and A_0 in (11) and (12) are evaluated by the formulas for B and A, but jE_0 should be substituted for $\overline{jE}$. The remaining notation is as before.

Let us analyze the results obtained and let us compare with the experimental data [2] which have been obtained for a GDC with the following geometric parameters: h = 0.034 m, s = 0.034·0.065 m^2, Δ = 0.02 m, q = 8. Moist air was used in [2]. One of the GDC operating modes was characterized by the following parameters: X_{H_2O} = 0.015, p_0 = 7800 Pa, static pressure drop over the length of the GDC ≈ 33 Pa for ΣI_i = 0.7 A. If it is considered that T_0 = 290°K, then v_0 = 86.45 m/sec. The power delivered to the gas was 15 kW·m^{-1}. A fraction of this power, equal to 0.11 here, went into local has heating (unrelated to the molecule vibrational energy relaxation). According to our model, this fraction is expressed by the terms in the square brackets in [2]. Therefore

$$1 - \xi_1 + \xi_2(U_a + U_K)/U = 0.11\,[1 + \xi_2(U_a + U_K)/U]. \tag{13}$$

It is determined in [2] that thermal power equal to ΣI_i·270 V, i.e., $1 - \xi_2 = 270/(U_k + U_a)$ is liberated at the electrodes. For a copper cathode U_k = 370 V [3] (p. 296); for a copper anode U_a = 0.5hp, where the dimensionality of the coefficient is V·Pa^{-1}·m^{-1} [4]. For the conditions of the GDC operating mode presented above we find ξ_2 = 0.463. The mean voltage on the electrodes was U = 15,000 Δ (q − 1)/ΣI_i + 270 = 3270 V. Taking this into account, we determine ξ_1 = 0.9288 ∿ 0.93 from (13). Hence it follows that ∿ 0.07 of the PC power density is expended in local heating. According to the estimates [2], the fraction of the electron energy lost in elastic collisions is around 0.002. Therefore, local gas heating in the PC of a discharge because of the excitation of molecule rotational degrees of freedom is 0.07/0.002 ≈ 35 times more efficient than the elastic electron energy scattering. This value agrees with the theoretical computation [12].

The results of computing nonequilibrium flow parameters in the channel of the experimental installation [2], executed by means of (9) and (10), are presented in Fig. 2. In calculating the coefficients A, B, C, D, λ, k_1 and k_2, the following data were used in addition to those presented above: μ = 28.8 kg/kmole, c = 1008 J/kg·K, $\overline{jE} = (U - U_k - U_a)\,\Sigma I_i/sq\,\Delta$ = 5.478·10^6 W/m^3, R = 8314 J/kmole·K. The dependence $\langle p\tau\rangle^{-1} = f(T)$ displayed in Fig. 1 was calculated with the molar fractions of those air gas components (see [13, p. 294]) taken into account for which we knew the reduced binary relaxation times for nitrogen. In our case these were: nitrogen, oxygen, water vapor, carbon dioxide, argon, helium, and hydrogen. Therefore, we determined $k_1 = f(T = 290)/290 = 5.937\cdot10^{-5}$ $Pa^{-1}\cdot sec^{-1}\cdot K^{-1}$; $k_2 = df(T \approx 320)/dT = 2.278\cdot10^{-4}$ $Pa^{-1}\cdot sec^{-1}\cdot K^{-1}$ and D = 0.0494 $Pa^{-1}\cdot sec^{-1}$. Assuming only nitrogen to be vibrationally excited, we take $\delta E = 3354\cdot1.38\cdot10^{-23}$ J. The accuracy of the computation using the formulas obtained can be judged by comparing the solid and dashed curves in Fig. 2. The latter are the results of a numerical computation of the complete system of one-dimensional gasdynamics equations on an electronic digital computer with gas friction on the channel wall taken into account (the friction coefficient was taken equal to 0.02), the equations of state and vibrational kinetics by the method elucidated in [1]. The coefficients characterizing

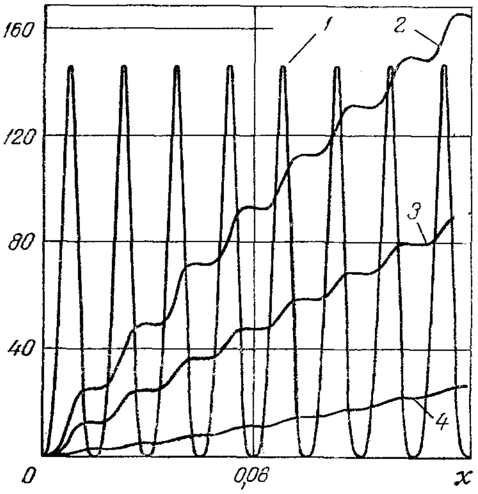

Fig. 3. Influence of actually existing modulation of the discharge power density on the flow parameters in a gas-discharge chamber: 1) $jE = jE_0 \sin^4(\pi x/\Delta)$, $10^5\ W \cdot m^{-3}$, 2) $N^{(1)}$, $10^{21}\ m^{-3}$, 3) $w^{(1)}$, $10^3\ J \cdot kg^{-1}$, 4) $(T^{(1)} - T_0)$, °K.

the local gas heating and excitation rates are here taken equal to those coefficients in the analytic computation. It is seen from Fig. 2 that in this case the first and second approximation formulas assure a flow parameter computation accuracy adequate for practice. To determine the limits of applicability of (9), a comparison is made between the flow parameters values obtained from (9) and the results of the digital computer computations for diminished gas velocities at the GDC input (it is here assumed that the power density in the discharge remained as before). The errors in the approximate formulas (9) are ~2, 12, 80% in w; ~5, 0, 40% in ΔT; ~1, 11, 80% in η; ~2, 11, 80% in N, ~1, 0, 16% in v from the values calculated on the digital computer, respectively, for v_0 = 86, 43, 21.5 m/sec. Here gas heating calculated on the digital computer was, respectively, 25, 73, 293°K for x = 0.16 m. Formulas (10) yield an error of magnitude smaller in these cases. An increase in the error as the gas temperature grows is related to the error in approximating the function f(T) by the line 3 as is clear from Fig. 1. To correct and check the error in such cases it is sufficient to separate the GDC into 2-3 sections in the computation and to apply (9) in each with a new value of the coefficient k_1.

Results of a computation of the nonequilibrium flow parameters in a GDC by means of (7) and (12) which take account of the actually existing modulation of jE with period Δ are presented in Fig. 3. It is seen that it results in flow parameter modulation but does not affect the flow parameters at the GDC output (compare with Fig. 2). Let us note that the modulation of jE in the near-anode region is expressed less sharply if a solid metal slab is the anode (as in [2]), and more sharply in the near-cathode region for sectioned cathodes (which corresponds to large values of the number m in the model utilized).

NOTATION

x, a gas flow coordinate; p(x), pressure; v(x), velocity; T(x), translational temperature; w(x), specific energy of gas molecule vibrations; R, universal gas constant; μ, molecular weight; c, specific heat at constant pressure; $\langle p\tau \rangle$, reduced vibrational relaxation time of the gas mixture; h(x), height (interelectrode gap); s(x), channel cross-sectional area; Δ, misalignment step in electrode series transverse to the flow; i, number of the electrode series transverse to the flow; q, number of such series along the flow; I_i, electrical current; U_i, voltage; U_{ai}, near-anode and U_{ki} is the near-cathode potential drop of the i-th series; U(x), $U_a(x)$, $U_k(x)$, j(x), functions modeling the voltage, anode and cathode potential drop, and electrical current density changes; jE(x), power density of the positive column of the discharge; ξ_1, fraction of this power expended in exciting molecule vibrations; ξ_2, fraction of the near-electrode powers expended in gas heating; p_0, v_0, T_0, s_0, w_0, same parameters at the GDC (section) input; δE, energy of one quantum of vibrational excitation; X_{H_2O}, molar fractions of water vapor; m, a natural number; π = 3.14 ... .

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